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Quantitative analysis of a smartphone pendulum beyond linear approximation: a lockdown practical homework

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We present a detailed analysis of a smartphone pendulum, part of which was given as a homework assignment to first-year undergraduate students. We took care in the design and construction of the pendulum itself to draw maximum benefit from the high quality of the embedded sensors. Our students build a pendulum and analyze their data using the damped harmonic oscillator model. We introduce them to residue analysis to make them aware of slight non-linearities in both the restoring and damping forces. Beyond what we ask our students, we present here results of numerical analyses to quantify these non-linearities and demonstrate that aerodynamic drag contributes quite significantly to damping. We finally discuss our pedagogical experience using this assignment in the classroom.

I. INTRODUCTION

Nowadays, almost every student possesses a smartphone that is equipped with sensors for purposes such as localization, orientation, gaming, photography etc. Apps that provide direct access to the sensor data have been put to use for practical teaching exercises for High School and University students.

Oscillatory motion is a keystone of physics and many papers have been published that use smartphones to revisit classical experiments¹ or propose innovative ped-agogical practices.^{2,3} Different configurations of simple pendulums⁴⁻⁶ or compound pendulums⁷ have been investigated. Other studies involve horizontal oscillating masses^{8,9} and, possibly, coupled systems.^{8,10} Motion is commonly monitored using the smartphone's accelerometers^{7,11} but other sensors such as magnetic fields,^{12,13} light intensity^{9,14} and rotation¹⁵ may be used. Furthermore, some apps allow for combined rotation and acceleration recording which gives interesting investigations.¹⁵ Finally, other open platforms such as Arduino⁷ or video recording¹⁶ have also been used. An exhaustive resource letter on mobile devices and sensors for physics teaching recently appeared in this journal.¹⁷

Many protocols found in the literature and, particularly videos on the web, are more demonstrative than quantitative. As a common example, the smartphone is simply suspended by hand using its own power supply cord. The experiment is then quite simple but, suffers from movement of the attachment point and stiffness in the power cord. An agreement between theory and measurement within a few percent is often considered as a good achievement. However, the embedded sensors are of high quality and so allow, as we shall see, for a more detailed analysis that reveals a much richer physical content.^{8,15,18–20}

We present below the project we have assigned to a first

year undergraduate class, which has been developed over two years. Students build their own pendulum (following provided instructions) and analyze their data using increasingly refined numerical techniques provided in the form of a Python program. Our purpose is to introduce our students to the scientific approach and, in particular, to the necessity of repeated cycles of experimentation and modeling to develop a complete scientific understanding of real systems.

The paper is organized as follows: in the first section, we present our setup, its particular dynamics and the experiments we carried out to show how simple and compound pendulums differ. Then in the second section, we perform a conventional curve fitting using the damped harmonic oscillator model. Despite a seemingly good agreement, residue analysis shows a significant discrepancy between the damped sine wave and actual data. We thus introduce, in the third section, less common data analysis methods that exhibit the nature of nonlinearities both in the amplitude and in the instantaneous frequency of the pendulum. In the last section we discuss how the project is actually given to our students and some of the related pedagogical aspects.

II. SETUP AND DYNAMICS

A. Setup

In order to restrict the motion to a vertical plane and minimize spurious oscillations, the smartphone is placed in an envelope suspended by two parallel threads^{4–6} (Fig. 1). We took care in this apparatus design to enable a precise quantitative comparison later on. A thin and almost inelastic sewing thread runs from A to D through the holes B and C made in the envelope. The thread is adjusted until AD and BC are parallel, then secured us-

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ing adhesive tape at points B and C. A wooden stick is inserted between B and C to stiffen the top of the envelope (Fig. 2a). At points A and D, the thread is wrapped around a ruler to precisely set AD = BC = 19.0 cm. The ruler is then clamped to a beam with AD and BC parallel to the ground. This achieves an almost perfect parallelogram ABCD with side length $L \sim 2m$, with well defined suspension points.



FIG. 1. (Color online) a) sketch and b) picture of our pendulum. The smartphone is inserted in an envelope suspended by two parallel 2 m-long almost inextensible threads. These threads are wound on a ruler to allow both precise geometry and easy clamping onto a building beam. The smartphone's overall motion is circular, but the body does not rotate around its center of mass. The envelope orientation remains parallel to its orientation at rest: each point within the envelope follows a circular path with radius L, but each with a different center e. g. A for B, D for C and O' for G.

B. Kinematics and dynamics

The system is analyzed in the laboratory frame with fixed x and y axes being respectively vertical and horizontal. The smartphone oscillates in the xy-plane. Its motion is perpendicular to its thinnest dimension which reduces aerodynamic friction.

Our pendulum differs from more common setups in which the smartphone is held by its power supply cord, which behave as compound pendulums. The velocity of the center of mass is perpendicular to the supporting thread; parallel to the rotating y-axis of the smartphone. But since the smartphone size and weight are not negligible, rotation of the smartphone around its center of mass must also be considered. This makes quantitative comparisons difficult.

In contrast, our design is a deformable parallelogram with quite different kinematics. During the motion, BC remains parallel to the ground by construction. The system is thus in a circular motion, but does not rotate about its own center of mass. The motion of all of the points of the system describe circles of the same radius

but with different centers (Fig. 1). At any time, all the points have the same velocity and acceleration as the center of mass G. The actual position of the sensor inside the smartphone makes no difference. We will see that the system dynamics reduce to those of a simple pendulum.

These system dynamics are easily obtained from an energetic analysis. We consider the system of total mass m made up of the envelope and the smartphone, but we neglect the contribution of the threads. The forces to be considered are: the weight $\mathbf{W} = m\mathbf{g}$ and friction force \mathbf{F} both applied to the center of mass G and the threads' tension \mathbf{T}_B and \mathbf{T}_C applied at the suspending points (Fig. 1). We will first assume a viscous friction force¹⁴ $\mathbf{F} = -\alpha \mathbf{v}_G$, with α to be determined experimentally and \mathbf{v}_G the center-of-mass velocity.

Since the object does not rotate and all points translate at the same speed, the kinetic energy is,

$$E_K = \frac{1}{2}mL^2\dot{\theta}^2.$$
 (1)

The work-energy theorem $\Delta E=\vec{F}\cdot\Delta\vec{d}$ in differential form gives:

$$\frac{dE_K}{dt} = (\mathbf{P} + \mathbf{F}) \cdot \mathbf{v}_G + \mathbf{T}_B \cdot \mathbf{v}_B + \mathbf{T}_C \cdot \mathbf{v}_C.$$
 (2)

As stated before, $\mathbf{v}_B = \mathbf{v}_C = \mathbf{v}_G$. For a circular motion, the velocity is perpendicular to the threads, so $\mathbf{T}_B \cdot \mathbf{v}_B = \mathbf{T}_C \cdot \mathbf{v}_C = 0$. Eq. (2) then gives:

$$mL^2\dot{\theta}\ddot{\theta} = -mgL\dot{\theta}\sin(\theta) - \alpha L^2\dot{\theta}^2.$$
 (3)

To first order in θ , $\sin(\theta) \simeq \theta$, and we obtain the usual damped harmonic oscillator equation:

$$\ddot{\theta} + \frac{\omega_0}{Q}\dot{\theta} + \omega_0^2\theta = 0, \qquad (4)$$

with the undamped angular frequency $\omega_0 = 2\pi f_0 = \sqrt{g/L}$ and quality factor $Q = m\omega_0/\alpha$. As we shall see below, we recorded the smartphone's linear acceleration \ddot{y} . Since $y = L\sin(\theta)$, we have to first order in θ :

$$\ddot{y} = L\ddot{\theta}$$
, (5)

so the linear acceleration mimics the angular acceleration.

Our design has a high quality factor $Q \sim 500$. The actual pseudo-frequency $f_0\sqrt{1-1/4Q^2}$ differs from f_0 by a few parts per million which is much less than the relative uncertainty on f_0 itself. The gravity constant g is precisely known in the vicinity of our lab²¹: $g = 9.80428 \text{ m} \text{ s}^{-2}$. The main uncertainty source is thus our measurement of the thread's length $L = 2040 \pm 1$ mm. We thus finally expect for the pseudo-frequency:

$$f_0 = 0.3490 \pm 0.0002 \text{ Hz.}$$
 (6)

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FIG. 2. (Color online) a) Picture showing the slight deformation when the smartphone is inserted despite the wooden stick inserted to stiffen the envelope. b) Comparison of the different values of the pseudo-frequency as determined by : f_0 (black: damped harmonic oscillator model), f_{FFT} (red: FFT of the recorded data points), f_{fit} (green: damped sine wave fit), f_{ZC} (blue: zero-crossing times linear regression). The residual plot (Fig. 6b) shows that the explanation for this inconsistency is the variation of the frequency over time.

C. Experiment

The accelerometer data is recorded using the app Phyphox developed at Aachen University.²² It provides bluetooth connection with a computer for easy data transfer and remote control of data acquisition. The pendulum is set in the xy-plane, a few degrees from its equilibrium position, using a sewing thread. We let the system settle down for a while and then burn or cut the thread to release the pendulum as smoothly as possible. Acquisition is started a few oscillations later (Fig. 3). For compar-



FIG. 3. (Color online) a) Raw data b) first few seconds of the record showing oversampling of the data allowing strong noise reduction through low pass filtering (red line).

ison with f_0 , we first perform a FFT on the raw data (black curve in Fig. 4b). This widespread algorithm is quite often misunderstood by beginner students and used as a black box. Moreover, it is not easy to make sense of the peak's width regarding frequency uncertainty. In particular, as we shall see below, the frequency actually drifts slightly during the motion. This effect contributes to peak broadening and is completely hidden using the Fourier Transform. Nevertheless, FFT provides an easy and quick rough estimate.

The fundamental peak is found at $f_{FFT} = 0.3486 \pm 0.0005$ Hz. The peak is slightly asymmetrical and undersampled so we have taken its Half Width at Half Maximum (HWHM) as a conservative value of the uncertainty. The measured f_{FFT} is consistent with f_0 obtained from the damped oscillator model (Eqs. (4) and (6)) at 1σ -level with a relative precision on the order of 0.15% (Fig. 2b). We thus have a good overall understanding of both the experimental and theoretical aspects of our system.

Before we proceed to a more detailed analysis, let us now present a simple and revealing experiment that we performed to demonstrate a particular feature of our setup (not proposed to our students).

D. Model confirmation

As stated in Sec. II B, the double thread suspension implies that the oscillation frequency is independent of both total mass and mass geometry. We first add in the envelope a heavy piece of brass. The total weight of the system is more than doubled without shifting the center of gravity much (Fig. 4a). Secondly, we inserted a slab of corrugated cardboard below the smartphone. This significantly lifts up the center of mass of the system by 75 mm, *i.e.* 3.5% of *L*, while leaving the total mass nearly constant. In both cases, as seen on Fig. 4b, the oscillation pseudo-frequency is hardly affected. Its very small decrease (~ 0.2%) is attributed to thread elongation (~ 4 mm) after the heavy brass piece has been inserted.



FIG. 4. (Color online) a) A piece of brass or cardboard is inserted in the envelope to change the total mass and mass geometry. b) Close view of the fundamental peak of FFT power density spectrum of \ddot{y} : smartphone alone (black), smartphone and brass piece (red), smartphone lifted up by a cardboard piece (blue). The slight decrease in frequency, is attributed to the elongation of the threads after the heavy brass piece has been inserted.

III. CURVE FITTING

While curve fitting was not required of our students in this project, we can gain some valuable information by using it.

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A. Model curve

Eq. (4) has a simple solution usually introduced in the first few years of undergraduate education:

$$\theta = \theta_0 e^{-t/2\tau} \cos\left(\omega_0(t - t_0)\right),\tag{7}$$

where we introduced the relaxation time of energy $\tau = Q/\omega_0$. θ_0 and t_0 depend on the initial conditions. From Eq. (5) the horizontal acceleration may be written:

$$\ddot{y} = a_{\mu_0} e^{-t/2\tau} \cos\left(\omega_0 (t - t_0)\right), \tag{8}$$

where we neglected the terms coming from the derivative of the exponential decay as $Q \gg 1$.

B. Best fit

We use the above damped sine wave to fit our data using OriginPro 2016 software. We get a seemingly satisfactory result (Fig. 5a) with good statistical estimators: reduced $\chi^2 < 0.005$ and $R^2 \simeq 0.97$. The program determines the pseudo-frequency with impressive precision $f_{fit} = 0.348413 \pm 1.0 \ 10^{-6}$ Hz. Moreover, the oscillation amplitude is correctly reproduced, except perhaps towards the very start and end of the recording. f_{fit} is consistent with our previous measurement f_{FFT} within the error bars but disagrees with the model prediction of f_0 (Fig. 2b). There is a contradiction and a more careful inspection of the data is required. We will use a less common, although very efficient technique: residual plots.



FIG. 5. (Color online) a) A few periods around 480 s: raw data (black dots) and damped sine fit (red line). Inset: full record. Agreement seems quite satisfactory. b) Residues from the fit showing a large and structured discrepancy between the data and the fitted curve.

C. Residues

Residues are the difference, for each data point, between the actual and the fitted values. Residues hence magnify the discrepancies that may not be clearly visible on full scale figures such as Fig. 5a. In principle, the fitted curve captures all the deterministic part of the data. Residues are expected to be randomly distributed and so plotting them provides a visual representation of the noise level of the data. But contrary to these expectations, the residues shown here in Fig. 5b exhibit a deterministic oscillatory shape with typical amplitude exceeding 10% of the maximum acceleration. So, our fitted curve is no good at all, despite its apparent agreement.

Close inspection of the beginning and the end of the record shows that the fitting curve slowly shifts from the data, demonstrating a kind of Moiré effect (Fig. 5b). The randomly selected few periods shown in Fig. 5a are in phase with the model only by chance.

As the damped sine wave model in Eq. 8 has a constant pseudo-period, we can conclude that the frequency of the measured data slowly drifts. The two techniques used so far cannot describe such an effect.

IV. MORE DETAILED ANALYSIS

A. Pseudo-period determination

Instead of the pseudo-frequency, we may alternatively determine the pseudo-period directly from the acceleration time series. The pseudo-period is best determined from the acceleration zero crossings times $t_0(n)$ where n is the rank index of the half period. Measurements between successive maxima or minima^{4,5,11} give worse results due to lower statistics (only one point per period is involved) and the greater effect of noise around a stationary point.

First of all, we remove a possible offset from the raw data: we discard a few points at both ends of the record to select an (almost) integer number of oscillation periods. Then we compute the mean value within this subset which has been extracted from the whole dataset.

We identify zero-crossing times by looking for a sign change between two neighbouring points. Due to a combination of noisy data with the high sampling rate, typically 200 Hz, the data may exhibit several sign changes around a single zero crossing. One solution is to use a low-pass filter to remove high frequency noise (see Fig. 3b). After the filter is applied to the data, there is only one sign change per zero crossing and so the zero crossing can be localized within half the sampling time (typically a few milliseconds *i.e.* a thousandth of the oscillation period.)

We also propose to our students a more advanced technique of accurately estimating zero-crossing times, interesting from a pedagogical point of view as it implements local approximation of a function. We extract from the dataset a sample of typically 20 data points on both sides of a sign change that we previously located using the first technique. We then perform a linear fit on this subset and compute a better approximation of the actual zero-crossing time from the regression parameters. This algorithm is thus quite effective as it enhances both noise filtering and time resolution.

Typical results for zero-crossing times are shown in Fig. 6a, together with a linear fit as expected for the

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damped oscillator model Eq. (8). At first sight, agree-



FIG. 6. (Color online) a) $t_0(n)$ are the zero-crossing times of \ddot{y} successively numbered by n (black points). Red line: linear adjustment hardly distinguished from data points. b) Residual plot showing that the oscillation period is not constant, but has a regular variation over time.

ment seems perfect, which is confirmed by the statistical estimators: reduced $\chi^2 < 0.01$ and R^2 rounded to 1 by the data analysis software. The pseudo-period is twice the slope. We get $T_{ZC} = 2.86868 \pm 4 \times 10^{-5}$ s from which we determine the pseudo-frequency f_{ZC} = $0.34859 \pm 0.5 \times 10^{-5}$ Hz. f_{ZC} is consistent with our two previous measurement f_{fit} and f_{FFT} but disagrees with the model prediction f_0 (Fig. 2b).

However, we find that the residual plot (Fig. 6b) clearly demonstrates that the data points are not described well by straight line, with the residual errors having a somewhat parabolic shape as a function of time, rather than the random scatter that would be expected if the underlying function was truly linear. The oscillation frequency is not constant, meaning that non-linear terms must be taken into account.

B. Amplitude variation

The damped harmonic oscillator model also predicts an exponential decay of the amplitude. The students test this property by plotting in log-scale the absolute value of the acceleration (Fig. 7a). Instead of the expected linearly decreasing envelope, we observe a concave shape with a typical deviation from the straight line by $\sim 15\%$ in the middle of the record. Since the decay is faster than expected at the beginning and then slows down, this implies that viscous damping (which is linearly proportional to velocity) is not the only dissipation mechanism in the experiment.

C. Numerical Simulations

For the sake of completeness, we have performed numerical simulations and supplied them in the Supplementary Material.²³ These are not discussed with our students. But we demonstrate here that (i) anharmonicity of the gravitational potential is not the main source of



FIG. 7. (Color online) a) Maximum of the absolute value of the acceleration for each half period (black dots, in log-scale). Exponential decay, as predicted by the damped harmonic os cillator model, would produce a linearly decreasing amplitude (red dashed line) which is obviously not the case. b) Numerical simulations, discussed in more detail in the Supplementary Material²³): data is quite well reproduced by an almost equal contribution of viscous and aerodynamic damping (red line).

the non-linearity of the restoring force, (ii) aerodynamic drag proportional to velocity squared contributes significantly to damping. In the end, our model reaches a 10^{-3} relative accuracy level over the whole recorded time series

V. PEDAGOGICAL ASPECTS

A. Project structure and objectives

The harmonic oscillator and harmonic approximation are widely taught in many areas of physics: mechanics, electricity, Drude-Lorentz model, IR spectroscopy, Debye model etc. We have developed the detailed analysis presented above over two years for use in a physics project given to first year undergraduate students. These students follow a selective two year course somewhat particular to the French education system which is significantly more intensive, especially in physics and mathematics, than our Undergraduate University Training.²

This project is given in the form of a comprehensive scientific investigation performed during a two-week holiday. The students are guided throughout their investigation process by a ten-page document. For data analysis purposes, a Python program that they are free to modify, is provided on a collaborative platform (see supplementary material for the document and program, in French). The program does not include part of the work presented here (Secs. IID, IIIB, IVC) but contains instead, a more typical phase portrait and energetic analysis not discussed here.

To keep the project attractive, it is not graded but students are required to write a report (typically 8 pages \log^{23} .) What we expect from our students is:

• the demonstration of the linearized equations of motion and energetic aspects,

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- \bullet the construction of their own pendulum,
- the data acquisition, basic use of the Python program and discussion within the harmonic framework,
- a first attempt at residue analysis and the log-scale method presented in Sec. IV B.

The project is designed to lead them to realize that the damped harmonic oscillator model presented in class is highly relevant and captures most of the physical phenomenon, but that it is not entirely exact. Regarding the explanation of the non-isochronism of the oscillations, we expect them to mention the linearization of the sine term, even if, as discussed in Sec. IV C, it might not be the major contribution in this type of setup. Concerning the non-exponential decay of the amplitude, we have no specific expectation as they have little knowledge about aerodynamics at this stage. Thus both of these issues are expected to be only qualitatively invoked by the students. Ideally, they may understand that high quality data may reveal some subtle effects if we are careful with the analysis, alternating model improvement and better data processing. We are quite aware that it is a very ambitious goal, and so far we have found that it is only partially achieved. Nevertheless, we want to awaken their curiosity and we think the amusing dimension of doing real physics with one's smartphone may help.

B. Discussion

We gave this project for the first time last year during the first lockdown due to the Covid-19 pandemic. The project was optional and conceived as a break during this very challenging period. Only half of the ~ 45 students took part. In those difficult times, the students who did not get involved said they were feeling too busy and had no time or energy to devote to what they perceived as extra work. The reduced interactions between themselves and with the teacher were also often put forward as reasons. These special circumstances also explain why half of the students who did participate carried out only part of the work, essentially the playful pendulum construction. However, most of the 10 students who completed the whole assignement found it an interesting experience and said they will reuse their smartphone for future experimental work. A few of them were even quite enthusiastic. These first promising results, despite adverse conditions, led us to reiterate.

This year, under less restrictive lockdown conditions, the project was compulsory. 8 students worked it alone and 36 worked in pairs. Half of the groups did the whole project together, the other half shared the tasks. We carried out a survey to identify their difficulties and appreciation of the project²³ (in French). Overall, only one group found this experiment rather negative compared to 30 positive and 12 quite positive. The main issues the participants reported were difficulties in understanding some algorithms and part of the theoretical aspects. Indeed, we did not include the full, derivation of Eq. (5), which uses several assumptions and linearization steps. They appreciated the teacher dedicating time to helpful discussions on these difficult points. Finally, after their reports had been evaluated and the survey completed, there was 2h-long teaching sequence devoted to more informal discussions with, here again, positive feedback.

Concerning our pedagogical objectives, half of the students suitably compared the different methods of period determination, but rarely interpreted them (especially taking into account the error bars.) A vast majority (> 30 students) found the residues analysis interesting and realized there are some non–linearities in the real system. A few of them proposed, though with no physical explanation, a v^2 damping.

Globally, we are satisfied with the practical, experimental and numerical aspects of the project. Our students took pleasure in doing the experiment themselves, with some of them displaying even more initiative in the data analysis part. Conversely, we have a more nuanced conclusion concerning our demanding conceptual targets. We have the feeling that most students perceived that the damped harmonic oscillator model is not the whole story, but without really understanding there is more interesting physics behind the discrepancies. Likewise, the use of refined data analysis methods and improved models is, most often, not given as a mandatory exercise for students. This is not surprising for first year students: this project is a learning exercise about how to take a careful scientific approach to non-linear physics, far beyond their learned skills at that age. However, the work they produced and, more importantly, their questions, were very encouraging. For the first time they built their own experiment and collected exhaustive data that they had explain, as it is and not as it should be. This is far from the usual practical work aimed at illustrating the formal course. As such, next year we will renew this experiment, further improving the program with extra comments to more explicitly teach the scientific approach to unexplained results, and adding more insight to the theoretical analysis.

As a final remark, let us mention that we were quite pleased that 20% of the students expressed regret that the program was completely provided to them. One student even suggested broadening the project by proposing they write their own code during their computer classes. This anecdote, though particularly rewarding, has certainly has no statistical relevance. However, the survey shows that a significant part of the class really engaged in the project, even though the advanced physical processes involved in the real system were only very vagely imagined by most. Consequently, we think that this project is useful for our first year students. It might also be used fruitfully for graduate students, who could perhaps more actively use their practical know-how and wider theoretical knowledge to throughally understand the real system.

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Our project aims at stimulating our students' curiosity by the fun use of their own smartphone. The high quality data that a smartphone provides can, in turn, lead the students to deeper investigations of the physical processes involved. A vast majority of our students enjoyed this

project.

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- [†] https://ires.univ-tlse3.fr/sim/
 ¹ see bibliography in *e.g.* "Relevant information about using a mobile phone acceleration sensor in physics experiments", J. Kuhn, Am. J. Phys. **82**, 93-94 (2014); doi: 10.1119/1.4831936.
- ² "Implementation of smartphone-based experimental exercises for physics courses at universities", A. Kaps, T. Splith and F. Stallmach, Phys. Educ. **56**, 035004 (2021).
- ³ "Manually driven harmonic oscillator", M. N. S. Silva and J. T. Carvalho-Neto, Phys. Educ. **56**, 035006 (2020).
- ⁴ "Analyzing simple pendulum phenomena with a smartphone acceleration sensor", J. Kuhn and P. Vogt, Phys. Teach. **50**, 439-440 (2012); doi: 10.1119/1.4752056.
- ⁵ "Analysis of pendulum period with an iPod touch/iPhone", J. Briggle, Phys. Educ. 48, 285-288 (2013).
- ⁶ "Study of the conservation of mechanical energy in the motion of a pendulum using a smartphone", T. Pierratos and H. M. Polatoglou, Phys. Educ. **53**, 015021 (2018).
- ⁷ "Physical pendulum experiment re-investigated with an accelerometer", C. Dauphin and F. Bouquet, Papers in Physics **10**, 100008 (2018).
- ⁸ "A quantitative analysis of coupled oscillations using mobile accelerometer sensors", J. C. Castro-Palacio *et al.*, Eur. J. Phys., **34** 737-744 (2013).
- ⁹ "Using a mobile phone acceleration sensor in physics experiments on free and damped harmonic oscillations ", J. C. Castro-Palacio *et al.*, Am. J. Phys. **81**, 472-475 (2013).
- (2019).
 ¹¹ "Probing higher-order approximations in large amplitude oscillations of a physical pendulum", S. Aiola *et al.*, Phys. Educ., 46, 247-249 (2011).
- ¹² "Measurement of g using a magnetic pendulum and a smartphone magnetometer", U. Pili, R. Violanda,

and Cl. Ceniza, Phys. Teach. ${\bf 56},\ 258\text{-}259$ (2018); doi: 10.1119/1.5028247.

- ¹³ Reference to AJP MS00003 to be published, "Instructional Laboratories and Demonstrations", [Details will be inserted by AIPP]
- ⁴ "Exploration of Large Pendulum Oscillations and Damping Using a Smartphone", D. Li, L. Liu, and S. Zhou, Teach. **58**, 634-636 (2020); doi: 10.1119/10.0002729.
- ¹⁵ "Frequency doubling in a pendulum", S. Reinhold and M. Ziese, Eur. J. Phys. 42, 025003 (2021).
- ¹⁶ "Study of oscillatory motion using smartphones and tracker software", A. Amoroso and M. Rinaudo, Journal of Physics: Conf. Series, **1076**, 012013 (2018); doi:10.1088/1742-6596/1076/1/012013.
- ¹⁷ Reference to AJP MS00848 to be published, "Resource Letter MDS-1: Mobile Devices and Sensors for Physics Teaching" [Details will be inserted by AIPP]
- ¹⁸ "The study of two-dimensional oscillations using a smartphone acceleration sensor: example of Lissajous curves", L. Tuset-Sanchis *et al.*, Phys. Educ. **50**, 580-586 (2015).
- ¹⁹ "Exploring phase space using smartphone acceleration and rotation sensors simultaneously", M. Monteiro, C. Cabeza and A. C. Martí, Eur. J. Phys. **35**, 045013 (2014); doi:10.1088/0143-0807/35/4/045013.
- ²⁰ "Une approche quantitative de la loi de Beer-Lambert avec un smartphone", R. Mathevet, *et al.*, Bulletin de l'Union des Professeurs de Physique et de Chimie (in French), **113**, 1079 (2019) and **113**, 1357 (2019).
- ²¹ https://infoterre.brgm.fr rapports RC-56265-FR.
- ²² https://phyphox.org/.
- ²³ See supplementary material at [URL will be inserted by AIPP], which contains description and code for numerical simulations, handouts given to students (in French), and some further data analysis.
- ²⁴ See supplementary material²³ for more details on these aspects of the French educative system.