Quantitative analysis of a smartphone pendulum beyond linear approximation: a lockdown practical homework

the authors
their affiliations

We present a detailed analysis of a smartphone pendulum, part of which being given as a practical homework to first year post-baccalaureate students. Some care is taken in the design and realization of the pendulum itself. This allows us to draw maximum benefit from the high quality data provided by the embedded sensors. Doing so, we make the student feel that the damped harmonic oscillator model is insufficient to account for the actual data. Beyond what we request from our students, we carry out here some numerical simulations to identify and quantify non-linearities in the restoring force and demonstrate that aerodynamic drag contribute quite significantly to damping. In the end, we reach a modeling of the system better than the 10^{-3} precision level. A discussion of our pedagogical experience is given at the end of the article.

I. INTRODUCTION

Nowadays, smartphones are quite widespread especially among students. For different purposes such as localization, orientation, gaming, photography..., they are equipped with a lot of sensors. Many Apps giving direct access to the raw data of those sensors are now available. Thus, the attractive or even entertaining aspects of smartphones have developed their use in high school and University practical teaching.

Oscillatory motion is a keystone of physics and many papers have been now published, revisiting classical experiments[1] or proposing innovative pedagogical practices[2,3]. Different configurations of simple pendulums[4–6] or compound pendulum[7] have been investigated. Other studies involve horizontal oscillating masses[8,9] and, possibly, coupled systems[8,10]. Motion is commonly monitored using the smartphone's accelerometers[7,11] but other sensors such as magnetic field[12], light intensity[9,13], rotation[14] may be used to record the motion. Furthermore, some apps allow for combined rotation and acceleration recording which gives interesting investigations[14]. Finally, other open platforms such as Arduino[7] or video recording[15] are also used.

Many protocols found in the literature and, particularly, videos on the web, are more demonstrative than really quantitative. As a common example, the smartphone is hand-held by its supply cord. The experiment is then quite simple but suffers from oscillations in the attachment point and stiffness in the suspending wire. From the recorded oscillatory motion a qualitative agreement within a few percent is often considered as a good achievement. However, despite their low cost, the embedded sensors are of high quality and allow, as we shall see, for a detailed analysis that reveals a much richer physical content[8,14,16–18].

We present below study of a pendulum project we have been giving for the second consecutive year to a first year post-baccalaureate class. Our purpose is to introduce them to the scientific approach and, in particular, to return trips between experimentation and modeling. Ideally we would like to introduce them to the subtle reversal that it is not the experiment that illustrates the theory but conversely, the model, within a given theoretical framework, that quantitatively interprets the data. To this end, after a conventional analysis using the damped harmonic oscillator model, we exhibit non-linearities in both frequency and amplitude decrease. We go however in this work further than what is presented to our students in several places, especially in identifying and quantifying the non-linearity sources with help of numerical simulations.

The paper is organised as follows. In the first section, we present our setup, its particular dynamics and the experiments we carried out to show how it differs from simple or compound pendulums (not given to students). Then, in the second section, we perform a conventional curve fitting analysis using a damped sine wave. Despite a seemingly overall very good agreement, residue analysis shows a significant discrepancy between the model and actual data. In the third section, we introduce less common data analysis methods to study more precisely both the amplitude and the instantaneous frequency of the pendulum. Finally, we carry out a numerical simulation of our system (not given to students). With moderate effort and simple concepts beyond the linear approximation, better than 10^{-3} relative precision and accuracy level over the full record is reached, showing that smartphones are not only playful for illustrating physics but quite efficient in doing quantitative physics. In the last section we discuss how the project is actually given to our students and some of the related pedagogical aspects.

II. SETUP AND DYNAMICS

A. Setup

In order to maintain the motion in a vertical plane and minimize spurious oscillations, the smartphone is placed in an envelope suspended by two parallel wires[4–6] (Fig. 1). We take some care in this first step for sake of quantitative comparison. A single thin and almost inelastic sewing wire runs from A to D through the holes B and C made in the envelope. This allows a precise adjustment of the parallelism of AD and BC which is then secured using adhesive tape to pin the wire at B and C. A wooden stick is then inserted between B and C to stiffen the top of the envelope (Fig. 6b). In A and D the wire is wrapped around a ruler to precisely set $AD = BC = 19.0$ cm. The ruler is then clamped to a beam with AD and BC parallel to ground. This achieves an almost perfect parallelogram $ABCD$ of length $L = 2036 \pm 1$ mm with well defined suspending points.

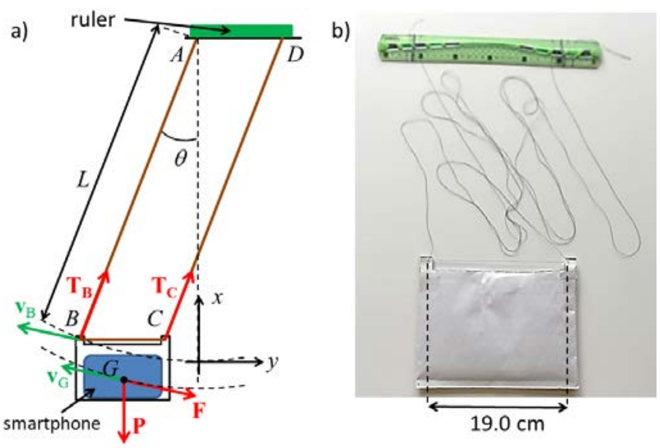

FIG. 1. (Color online) a) sketch and b) picture of our pendulum. The smartphone is inserted in an envelope suspended by two parallel $L \sim 2$ m-long almost inextensible wires. These wires are wound on a ruler to allow both precise geometry and easy clamping on a building beam.

B. Kinematics and dynamics

The system is naturally studied in the laboratory frame with x and y axes being respectively vertical and horizontal using the smartphone convention. Contrary to some investigations cited before, we will study the oscillations in the xy-plane to minimize aerodynamic drag. We have a deformable parallelogram with a peculiar kinematics. During the motion, BC remains by construction parallel to ground. The system is thus in circular translation and not in rotation around a suspending point as for usual pendulums. All points of the system describe circles of the same radius L with different centers. At any time, they all share the same velocity and acceleration and the actual position of the sensor inside the smartphone makes no difference.

The kinematics of any point of the envelope chosen as the origin when at rest reads:

$$x = L\,(1 - \cos(\theta)) \qquad y = L\sin(\theta), \tag{1}$$

from which the linear accelerations can easily be evaluated:

$$\ddot{x} = L\left(\cos(\theta)\,\dot{\theta}^2 + \sin(\theta)\,\ddot{\theta}\right)$$
$$\ddot{y} = L\left(-\sin(\theta)\,\dot{\theta}^2 + \cos(\theta)\,\ddot{\theta}\right), \tag{2}$$

where the dot sign denotes time derivative. In typical situations presented below, the amplitude of oscillation does not exceed 0.1 rad and a series expansion of Eq. 2 seems appropriate. Both terms in the vertical acceleration $\ddot{x}$ are of second order and will be discarded as well as the first term of the vertical component which is of third order. At first order, only remains:

$$\ddot{y} = L\ddot{\theta}, \tag{3}$$

that is to say the linear acceleration is proportional to the angular acceleration. Our study will be restricted to the horizontal acceleration $\ddot{y}$.

The dynamics is easily obtained from an energetic point of view. We consider the system of total mass m made up of the envelope and the smartphone and neglect the wires' contributions. The forces to be considered are: the weight $\mathbf{P} = m\mathbf{g}$ and friction force $\mathbf{F}$ both applied to the center of mass G and the wires' tensions $\mathbf{T}_B$ and $\mathbf{T}_C$ at the suspending points (Fig. 1). We will first assume a viscous friction force[13] $\mathbf{F} = -\alpha\mathbf{v}_G$, with α to be determined experimentally and $\mathbf{v}_G$ the center-of-mass velocity.

From König's theorem, the kinetic energy of the system is $E_K = E^* + 1/2\,mv_G^2$ where the kinetic energy in the center-of-mass frame is here $E^* = 0$. We have thus:

$$E_K = \frac{1}{2}mL^2\dot{\theta}^2. \tag{4}$$

The work-energy theorem reads:

$$\frac{dE_K}{dt} = (\mathbf{P} + \mathbf{F}).\mathbf{v}_G + \mathbf{T}_B.\mathbf{v}_B + \mathbf{T}_C.\mathbf{v}_C. \tag{5}$$

For a circular motion the velocity is perpendicular to the wires so $\mathbf{T}_B.\mathbf{v}_B = \mathbf{T}_C.\mathbf{v}_C = 0$. Eq. 5 then gives:

$$mL^2\dot{\theta}\ddot{\theta} = -mgL\dot{\theta}\sin(\theta) - \alpha L^2\dot{\theta}^2. \tag{6}$$

At first order $\sin(\theta) \simeq \theta$ and we finally get the usual damped harmonic oscillator equation:

$$\ddot{\theta} + \frac{\omega_0}{Q}\dot{\theta} + \omega_0^2\theta = 0, \tag{7}$$

with the undamped angular frequency $\omega_0 = 2\pi f_0 = \sqrt{g/L}$ and quality factor $Q = m\omega_0/\alpha$.

C. Experiment

The accelerometer data is recorded using the App phyphox developed at Aachen University[19]. It allows remote control of the smartphone and easy data transfer. The pendulum is set in the xy-plane, a few degrees from its equilibrium position by a sewing wire. We let the system settle down for a while and then burn the wire to release the pendulum as smoothly as possible. Acquisition started a few oscillations later (Fig. 2). Our design

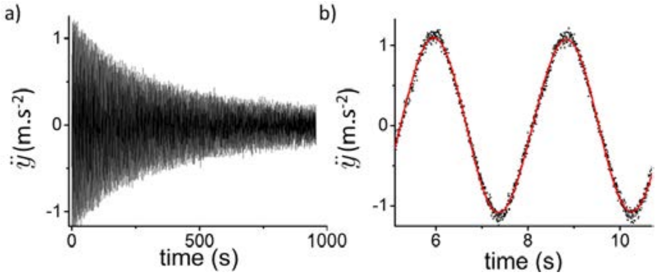

FIG. 2. (Color online) a) Raw data b) first few seconds of the record showing oversampling of the data allowing strong noise reduction through low pass filtering (red line).

has a high quality factor $Q \sim 500$ so that the actual pseudo-frequency $f_0\sqrt{1 - 1/4Q^2}$ differs from f_0 by a few parts per million which is much less than the relative uncertainty on f_0 itself mainly due to our measurement of the length L as $g = 9.80428 \pm 5\,10^{-6}$ m.s^{-2} is precisely known in the vicinity of our lab[20]. We thus expect a pseudo-frequency:

$$f_0 = 0.34925 \pm 0.00017 \text{ Hz}. \tag{8}$$

As a first estimate of the experimental pseudo-frequency we perform a FFT on the raw data (black curve in Fig. 3b). The fundamental peak center frequency is $f_{FFT} = 0.3486 \pm 0.0005$ Hz. The peak is slightly asymmetrical and under-sampled so we take its Half Width at Half Maximum (HWHM) as a conservative estimate of the uncertainty. The expected pseudo-frequency f_0 from the damped oscillator model Eq. 7 is thus compatible with the measured frequency f_{FFT} at 1σ level with a relative precision on the order of 0.15% (Fig. 6a).

D. Model confirmation

As stated before (Sec. II B), the double wire suspension implies that the oscillation frequency is independent of both total mass and mass geometry. We tested these predictions adding first in the envelope an heavy piece of brass. The total weight of the system is more than doubled without moving much the center of gravity (Fig. 3a). Secondly, we inserted a slab of corrugated cardboard that significantly lifted up the center of mass of the system by 75 mm, *i.e.* 3.5% of L, almost unchanging the total mass. In both cases, as seen on Fig. 3b, the oscillation pseudo-frequency is hardly affected. The very slight decrease of oscillation frequency ($\sim 0.2\%$) is attributed to wire elongation ($\sim$ 4 mm) after the heavy brass piece has been inserted.

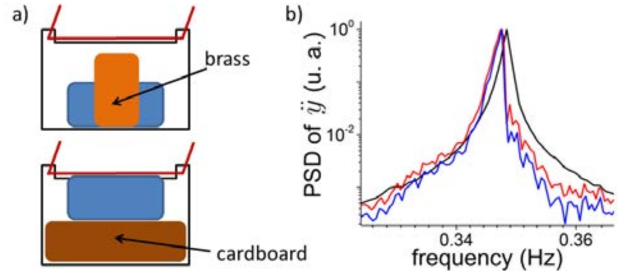

FIG. 3. (Color online) a) A piece of brass or cardboard is inserted in the envelope to change the total mass and mass geometry. b) Close view of the fundamental peak of FFT power density spectrum of $\ddot{y}$: smartphone alone (black), smartphone and brass piece (red), smartphone lifted up by a cardboard piece (blue). The slight decrease in frequency, is attributed to wires elongation after the heavy brass piece has been inserted.

III. CURVE FITTING

A. Model curve

The damped harmonic oscillator model (Eq. 7) is the generic first order motion about any rest point. It has an easy solution usually introduced in the first undergraduate years:

$$\theta = \theta_0\, e^{-t/2\tau} \cos\left(\omega_0(t - t_0)\right), \tag{9}$$

where we identified as before the angular pseudo-frequency to the undamped one and introduced the relaxation time of energy $\tau = Q/\omega_0$. θ_0 and t_0 depend on initial conditions. From Eq. 3 the horizontal acceleration may be written:

$$\ddot{y} = a_{y_0}\, e^{-t/2\tau} \cos\left(\omega_0(t - t_0)\right), \tag{10}$$

where we neglected the terms coming from the derivative of the exponential decay as $Q \gg 1$.

B. Best fit

A popular tool to analyse such data is curve fitting. It is however a complex procedure based on sophisticated algorithms most commonly used as a black box performing well. When applied to our data using OriginPro 2016 software we get a seemingly satisfactory fit (Fig. 4a) with good statistical estimators: reduced $\chi^2 < 0.005$ and $R^2 \simeq 0.97$. The pseudo-frequency is determined with seemingly impressive precision $f_{fit} = 0.348413 \pm 1.0\ 10^{-6}$ Hz and the amplitudes match well together excepted perhaps towards the very start and the end of the recording. So far, so good. However, f_{fit} is no longer compatible with the damped harmonic oscillator model (Eq. 8 and Fig. 6a). A more careful inspection of data is required.

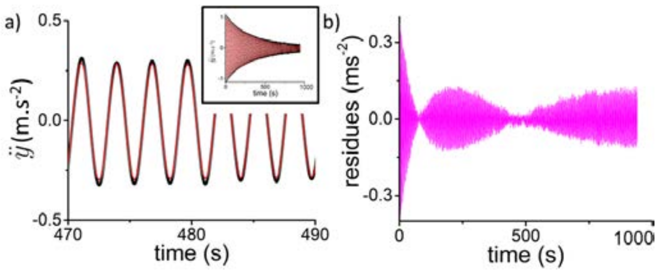

FIG. 4. (Color online) a) A few periods around 480 s: raw data (black dots) and damped sine fit (red line). Inset: full record. Agreement seem quite satisfactory. b) Residues from the fit showing a large and structured discrepancy between the data and the fitted curve.

C. Residues

Residual plots are very efficient to check the quality of a fitted curve. We use them systematically in the following.

Residues are the difference for each point between the actual data and the fitted value. They thus magnify the disparities not clearly visible on full scale figures such as Fig. 4a. The clear-cut result is shown in Fig. 4b: residues exhibit a deterministic oscillatory shape with typical amplitude exceeding 10% of the maximum acceleration. Thus, the fit is no good at all.

Inspection of the beginning and the end of the record shows that the fitting curve slowly shifts from the data resulting in a kind of Moiré effect (Fig. 4b). As the damped sine has a constant pseudo-period we can conclude that the frequency of the data slowly drifts: actual oscillations are not isochronal.

IV. MORE DETAILED ANALYSIS

A. Pseudo-period determination

The pseudo-period is best determined pointing the times of the acceleration zero crossings denoted by $t_0(n)$ where n is the rank label. Measurements between highs or lows[4,5,11] give worse results due to lower statistics (only one point per period is involved) and poor pointing around a stationary point.

First of all, we remove a possible offset from the raw data: we discard a few points at both ends of the record to select an (almost) integer number of oscillation periods. Then we compute the mean value of this subset which is subtracted to the whole dataset.

A nice feature of the accelerometers we haven't focused on yet is their high sampling rate, typically 200 Hz. The data from our 3s-period pendulum is highly over sampled which allows noise reduction by low pass filtering (see Fig. 2b). It greatly simplifies the procedure as each zero-crossing now happens between a unique pair of points. Our algorithm, implemented in Mathematica but easily transposed in other languages such as Python, proceeds as follows. Firstly, the zero-crossing are roughly located testing a sign change between two neighbouring points. Then a sample of typically 20 data points on both sides is extracted from the whole dataset on which we perform a linear fit. The zero-crossing time is then easily and precisely calculated from the regression parameters. The result is shown in Fig. 5a together with a linear adjustment as expected for the damped oscillator model (Eq. 10). At first sight the linear fit seems

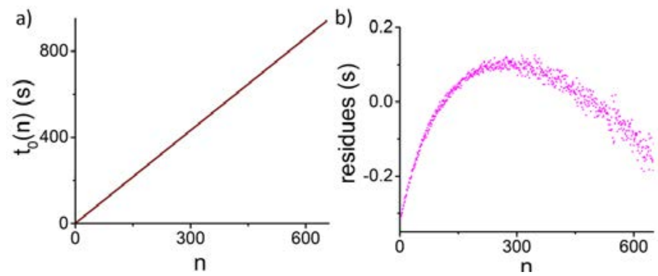

FIG. 5. (Color online) a) $t_0(n)$ are the zero-crossing times of $\ddot{y}$ successively numbered by n (black points). Red line: linear adjustment hardly distinguished from data points. b) Residual plot showing a significant departure of the oscillations from isochronism.

perfect which is confirmed by the statistical estimators: reduced $\chi^2 < 0.01$ and R^2 rounded to 1 by the data analysis software. The pseudo-period is twice the slope. We get $T_{ZC} = 2.86868 \pm 4\,10^{-5}$ s from which we determine the pseudo-frequency $f_{ZC} = 0.34859 \pm 0.5\,10^{-5}$ Hz. It agrees within the error bars with the FFT value and the damped sine fit but not with the damped harmonic oscillator model (Fig. 6a). However, here again, the residual

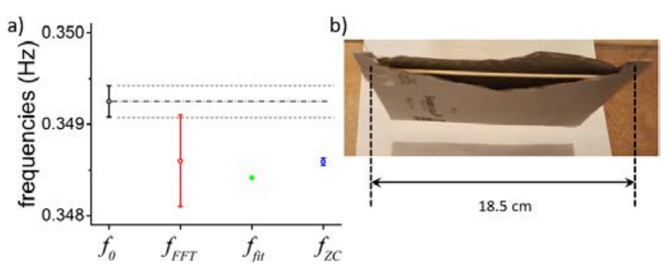

FIG. 6. (Color online) a) Comparison of the different values of the pseudo-frequency as determined by : f_0 (black), damped harmonic oscillator model, f_{FFT} (red), FFT of the recorded data points, f_{fit} (green), damped sine wave fit, f_{ZC} (blue), zero-crossing times linear regression. The neighbouring residue plot shows that the motion is actually non periodical, the different methods have no reason to give consistent values. b) Picture showing the slight deformation when the smartphone is inserted despite wooden stick inserted to stiffen the envelope.

plot (Fig. 5b) clearly proves that the data points significantly deviate from the straight line with a deterministic parabola-like distribution. Non-linear terms in the oscillation frequency must be taken into account.

B. Amplitude variation

The damped harmonic oscillator model also predicts an exponential decay of the amplitude. A simple and very demonstrative test of this property is the plot in log scale of the absolute value of the centered dataset (Fig. 7a). The damped oscillator model predicts an exponential decrease of the amplitude which manifests itself as a linear curve envelope. As seen in Fig. 7, it is clearly not the case. The typical discrepancy is on the order of 15% in the middle of the record.

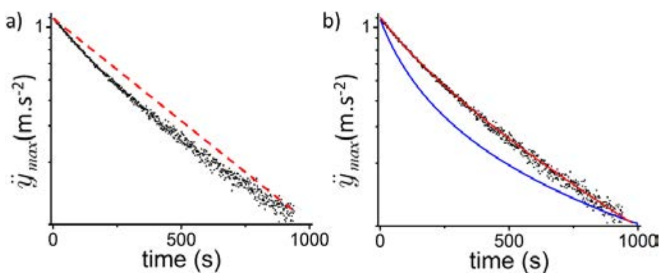

FIG. 7. (Color online) a) Maximum of the absolute value of the acceleration for each half period (black dots, in log scale). Exponential decay, as predicted by the damped harmonic oscillator model, would produce a linearly decreasing amplitude (red dashed line) which is obviously not the case. b) Numerical simulations: aerodynamic drag only (blue line) does not fit the data which is quite well reproduced by an almost equal contribution of viscous and aerodynamic damping (red line).

As a conclusion, the simple damped harmonic oscillator model does not reproduce neither the amplitude nor the pseudo-period drift of the data faithfully at the precision level provided by the sensors.

Unfortunately, taking into account anharmonicity and other forms of damping involve, in the best case special functions, but, most often, no analytical solution exists. We thus must resort to numerical simulations.

V. NUMERICAL SIMULATIONS

A. Amplitude variation

Inspection of Fig. 7a shows that compared to viscous friction, the actual damping is stronger at the beginning and weaker at the end of the record. Dissipation increases faster with the amplitude than viscous damping. We thus replace the viscous damping term of Eq. 7 by aerodynamic drag which is quadratic in the velocity and modeled as $-\beta\,\mathrm{sign}(\dot{\theta})\,\dot{\theta}^2$. Indeed, the typical speed of the envelope is $U = 0.2$ m.s^{-1}, its size $D = 0.1$ m and the kinematic viscosity of air $\nu = 16\,10^{-6}\mathrm{m}^2.\mathrm{s}^{-1}$. The associated Reynolds number is thus $Re = UD/\nu \simeq 6250 \gg 1$. Depending on the actual shape of the moving system, the air flow around the envelope might be turbulent. The simulated motion with such a quadratic damping is plotted in blue in Fig. 7b. Clearly, the shape is now too concave: the amplitude decreases too fast. Then, we can make the assumption that the highly contoured shape of the envelope is such that the turbulence is not fully developed. We thus reintroduce a viscous term $-\alpha\,\dot{y}$ in the model. Fitting the data with α and β as free parameters in the equation of motion is not an easily automated task. So we made a few trial and error and readily obtained the red curve in Fig. 7b which exhibit quite a satisfactory agreement with actual data (black dots).

α and β having different dimensions, we cannot compare them directly to evaluate the relative contributions of the linear and quadratic terms to damping. Numerical simulation proves to be very useful here. Multiplying each damping term by the velocity $L\dot{\theta}$ one gets the associated dissipated power which, in turn, can be numerically integrated to compute the energy losses. We find here that the aerodynamic drag contributes to 45% of the total energy dissipation. Naturally, this fraction is relative to our particular experiment and has no universal meaning. However, we can conclude that the aerodynamic drag is significant in many pendulum experiments carried out with a smartphone.

B. Anharmonicity

Despite the small angles involved in the experiment, the great quality of the sensors shows the anharmonicity of the motion unambiguously (see also large amplitude studies[11,13]). In the numerical simulation, it is naturally easy to replace the linearized restoring force by the exact one *i. e.* replace the $\omega_0^2\theta$ term in Eq. 7 by $\omega_0^2\sin\theta$. In our case the result is quite unsatisfactory. To fit the residues of Fig. 5b and thus account for the actual anharmonicity of the oscillations, we have to artificially multiply by ~ 6 the non linearity introduced in the model by the $\sin\theta$ term (black curve in Fig. 8a).

We checked that this 6-fold increase in the observed non linearity is not a numerical artifact the following way. The first order correction of the frequency f_0 of a simple pendulum with respect to its amplitude θ_0 is given by the Borda formula[11,13]:

$$f \simeq f_0(1 - \theta_0^2/16). \qquad (11)$$

As a first estimate, we can replace in Eq. 11 θ_0 by $\theta_0 e^{-t/2\tau}$. We get a model with both varying frequency and amplitude. The zero-crossing times can be calculated and residues extracted following the procedure presented before. The result, not shown here, compares well with the numerical simulation without 6-fold magnification of the non linearity. This validates our whole procedure and demonstrates thus there are extra non linearity sources

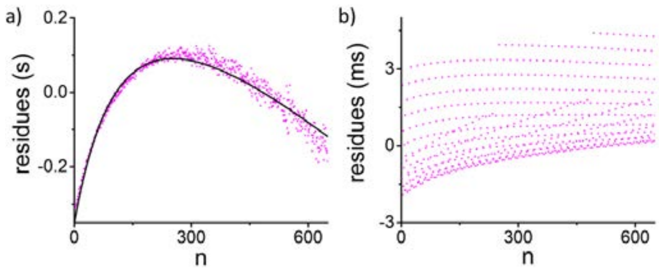

FIG. 8. (Color online) a) Comparison of anaharmonicity of our pendulum as deduced from the residues (magenta dots) of the zero-crossing times of the acceleration to a linear fit (Fig. 5) with an artificially 6-fold increase of the non linearity induced by the $\sin\theta$ term in the actual restoring force (black curve). b) Residues of the zero-crossing times of the acceleration compared to the full simulation with viscous and aerodynamic damping plus 6-fold increased non-linearity. Notice vertical scale in ms. The apparent structure in the residues come from numerical aliasing.

in our setup. For symmetry reasons ($\theta \leftrightarrow -\theta$), any non linearity manifest itself, at lower order, as θ_0^2 corrections and sums up with the Borda term.

We have no definitive explanation of the origin of the observed excess anharmonicity. The effect is undoubtedly significant but quite small, typically a few tens of milliseconds after one hundred periods of $\sim$ 3 s each. Our best hypothesis is the slight deformation of the envelope due to the smartphone weight despite the wooden stiffening stick we added (Fig. 6b). Distance BC is actually reduced to $\sim$ 18.5 cm compared to the designed AD = 19.0 cm. The idealized kinematics presented in Sec. II B no longer holds at the 10^{-2} relative precision level, which induces 10^{-4} corrections and amounts to a few tens of milliseconds over a few hundreds of seconds. This explanation is plausible and is of the right order of magnitude. However, there are many other possible imperfections that could contribute as well. This requires further investigations far beyond the scope of our study.

In the end, with viscous and aerodynamic damping together and a 6-fold increased non linearity, our model accounts for the data much better. Residue analysis shows that the $\sim$ 650 zero crossing times are predicted with a typical accuracy of 3 ms to be compared with the $\sim$ 3 s oscillation period of the pendulum (Fig. 8b).

VI. PEDAGOGICAL ASPECTS

We have developed the detailed analysis presented above on the occasion of a physics project we have been experimenting for two years now with a class of first year post-baccalaureate students. They follow a selective two year course called *Classe préparatoire aux Grandes Écoles* somewhat particular to the french educative system, significantly more intensive, especially in physics and mathematics, than our undergraduate University training. The harmonic oscillator and harmonic approximation are widely taught as transverse models in many areas of physics: mechanics, electricity, Drude-Lorentz model, IR spectroscopy, Debye model...

The project is presented in the form of a comprehensive scientific approach during a two weeks holidays. The work is divided in four parts: (i) theoretical modeling (ii) pendulum realization (iii) data analysis (iv) critical evaluation of the results. The students are guided throughout their own investigation process by a ten pages document and, for data analysis purposes, a Python program they are free to modify is provided on a collaborative platform (see Supplementary material). The program does not include the numerical simulations we show in Sec. V which we do not think adapted to first year students. This part is intended to higher education students and teachers, or possibly, as a support for a long term experimental project such as our *Travaux d'Initiative Personnelle Encadrés* (2h/week over the whole year). Incidentally, the supplied program contains instead a usual phase portrait and energetic analysis not discussed here.

What we expect from our students is (i) demonstration of the linearized equations of motion and energetic aspects (ii) built of their own DIY pendulum (iii) data acquisition, basic use of the Python program and discussion within the harmonic framework (iv) first initiation to residues analysis and the log scale method presented in Sec. IV. Ideally, we would like them realize that the damped harmonic oscillator model presented in class is highly relevant and capturing most of the physical phenomenon but that high quality data may reveal some subtle effects if we take care in the analysis, alternating model improvement and better data processing. We are quite aware that it is a very ambitious goal, only partially achieved as we shall see. Nevertheless, we want to awaken their curiosity, the younger the better, and the funny dimension of doing real physics with one's own smartphone may help. As for the explanation of the non-isochronism of the oscillations, we expect they mention the linearization of the sine term, even if, as we have shown, it is presumably not the major contribution in our kind of setups. Concerning the non-exponential decay of the amplitude, we have no specific expectation as they have few notions in aerodynamics yet. Both features are expected to be only qualitatively invoked by the students. To keep the project pleasant and attractive, it is not formally rated but students are requested a report (typically 8 pages long, see an example in Supplementary material).

We gave this project for the first time last year during the first lockdown due to covid-19 epidemics. It was optional and conceived as a break during this very challenging period. Only half of the $\sim$ 45 students took part to it. In this harsh time, they said feeling too busy and having no time or energy to devote to what they perceived as extra work. The reduced interactions between them and with the teacher was also often put forward.

These special circumstances also explain why half of the participants made only part of the work, essentially the playful pendulum conception. However, most of the 10 students who completed the job found it an interesting experience and said they will reuse their smartphone for future experimental work. A few of them were quite enthusiastic. These first promising results, despite adverse conditions, led us to follow up.

This year, under less restrictive lockdown conditions, the project was compulsory. 8 students made it alone and 36 working in pairs, with half of the groups doing the whole project together, the other half sharing the tasks. We had 26 (= 8+36/2) responses to the survey we made to identify their difficulties and appreciation of the project (see Supplementary Material). Overall, only one group found this experiment rather negative compared to 30 and 12 students who appreciated it positive or quite positive. The main issues they report are difficulties in understanding some algorithms and part of the theoretical aspects. Indeed, we have no correct full derivation of Eq. 3 which implies several assumptions and linearisation steps. They appreciated the teacher time dedicated this year to helpful discussions of these difficult points. A 2h-long teaching sequence has been devoted, after their reports evaluated and the survey completed, to more informal discussions with, here again, rather positive feedbacks.

Concerning our pedagogical objectives, half of the students suitably compared the different methods of period determination but rarely interpreted them taking into account the error bars. A vast majority (> 30 students) found the residues analysis interesting and realized there are some non linearities. A few of them proposed, with no physical explanation however, a v^2 damping.

Globally, we are very satisfied of the practical and experimental facets. Our students took pleasure in doing the experiment themselves with some of them demanding even more initiative in the data analysis part. Conversely, we have a more nuanced opinion concerning our demanding conceptual targets. We have the feeling most students perceived that the damped harmonic oscillator model is not the whole story without really understanding there is some interesting physics to do beyond. Likewise, the use of refined data analysis methods and improved models is most often not properly appreciated and conceived as a mandatory exercise. All this is not surprising for first year students: this project is a discovery and initiation to a real scientific approach not actual teaching of non-linear physics, far beyond their skills at that age. However, the work they produced and, more importantly their questionings, are very encouraging. For the first time they built their own experiment and were faced to exhaustive data they have to account for, as it is and not as it should be. This is far from usual practical work aimed at illustrating the formal course. In that respect, next year, we will renew this experiment improving the program with extra comments underlining more explicitly the scientific approach and adding a bit more help on the theoretical part.

As a final remark, let us indicate we were quite pleased that 20% of the students regretted the program was completely written and that one even suggested to widen the project proposing they write their own code during their computer classes. This personal testimony, particularly rewarding, has for sure no statistical relevance. However the survey shows that a significant part of the class really engaged in our proposal even if the advanced physical processes involved are only envisioned. We thus think this project useful for our first year students but also, it could be used fruitfully with graduate students who could actively reinvest their know-how and wider knowledge.

VII. CONCLUSION

In this article we have presented a detailed analysis of a smartphone pendulum experiment that supports and extends a project we give to our first year post-baccalaureate students. With reasonable care taken in the realization of the pendulum itself and some attention paid to data analysis, we show that the damped oscillator model is inconsistent with the high quality data available: isochronism of the oscillations, even at as low amplitudes as a few degrees, is clearly not satisfied. We further explain, in a more advanced part not suitable for beginner physicists, that the non linearity of the gravitational restoring force is not the leading contribution to non isochronism in our system and that viscous damping represents only part of dissipation.

By no means our study intend to depreciate the damped harmonic oscillator model whose physical and pedagogical values are priceless and constitutive to any physicist's initial education. Our project aims at stimulating student's curiosity by playful use of their own smartphone which in turn, by the high quality that they provide, can lead them to deeper investigations of the physical processes involved.

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